Vector Concepts in ℝⁿ – Summary Sheet

# 🧭 What is a Vector?

A vector is a quantity that has both:  
• Magnitude (size/length)  
• Direction  
  
It differs from a scalar, which only has magnitude (e.g., temperature, mass).

# 📌 Examples of Vectors in Real Life

• Velocity: 60 km/h to the North  
• Force: 10 N at 30° angle  
• Displacement: 5 meters to the right

# 🧮 Vector Representation

1. As an Arrow:  
 • A vector is like an arrow from point A to point B.  
 • Length represents magnitude; arrowhead shows direction.  
  
2. As Coordinates:  
 • Example: v = [2, -1, 3]ᵀ is a 3D vector with x, y, z components.  
 • Can be represented as a row [x₁, ..., xₙ] or column [x₁, ..., xₙ]ᵀ.

# 🧰 Vector Operations

➕ Vector Addition:  
 (x₁, ..., xₙ) + (y₁, ..., yₙ) = (x₁ + y₁, ..., xₙ + yₙ)  
  
🔁 Scalar Multiplication:  
 λ(x₁, ..., xₙ) = (λx₁, ..., λxₙ), ∀λ ∈ ℝ  
  
📐 Dot Product:  
 a • b = a₁b₁ + a₂b₂ + ... + aₙbₙ (returns a scalar)  
  
🔄 Cross Product (3D only):  
 a × b = c (vector perpendicular to both a and b)

# 🔍 Special Vectors

• Zero Vector: [0, 0, ..., 0] – No direction or magnitude  
• Unit Vector: Magnitude = 1, used to indicate direction  
• Basis Vectors: Standard vectors (î, ĵ, k̂) in 3D  
• Column Vector: Vertical layout (common in linear algebra)  
• Row Vector: Horizontal layout

# 🧠 Applications of Vectors

• Physics: Forces, velocity, acceleration  
• Computer Graphics: 3D transformations, lighting  
• Machine Learning: Weight vectors, gradient descent  
• Robotics: Motion planning, kinematics  
• Economics: Preference or portfolio vectors

# ✅ Summary

• Vectors have both magnitude and direction  
• Represented as tuples or arrays in ℝⁿ  
• Support addition, scaling, dot/cross products  
• Widely used in mathematics, science, and engineering